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Course: MPZ 3231-Engineering Mathematics IA **Model Answer No·02**

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1. a)

$$\cos y + (1+e^{-x})\sin y\,\frac{dy}{dx} = 0$$

$$\int \tan y\; dy = -\int \frac{1}{(1+e^{-x})}dx$$

$$-\int \frac{-\sin y}{\cos y}\; dy = -\int \frac{e^{x}}{(1+e^{x})}dx$$

$$\ln|\cos y| = \ln(1+e^{x}) + \ln c$$

$$\ln|\cos y| = \ln\; c(1+e^{x})$$

$$\therefore \cos y = c(1+e^{x})$$

When $x = 0 \quad y = \frac{\pi}{4}$

$$\therefore \frac{1}{\sqrt{2}} = 2c$$

$$\therefore c = \frac{1}{2\sqrt{2}}$$

$$\therefore 2\sqrt{2}\cos y = (1+e^{x})$$

b)

$$x^2(y+1) + y^2(x-1)\frac{dy}{dx} = 0$$

$$\int \frac{x^2}{(1-x)}dx = \int \frac{y^2}{(y+1)}dy$$

$$\int \frac{x^2}{(1-x)}dx = \int \frac{y^2}{(y+1)}dy$$

$$\int -\left[(x+1) + \frac{1}{(1-x)}\right]dx = \int \left[(y-1) + \frac{1}{(y+1)}\right]dy$$

$$-\frac{x^2}{2} - x - \ln|1-x| \; = \frac{y^2}{2} - y - \ln|y+1| \; + c$$

$$x^2 + y^2 + 2x - 2y + \ln\frac{(1-x)^2}{(1+y)^2} + c_1 = 0$$

c) **Method 1**

$$(xy + y^2) + (x^2 - xy)\frac{dy}{dx} = 0$$

Let $v = {}^{x}\!/_{y}$

$$\therefore x = vy$$

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$(xy + y^2)\left(v + y\frac{dv}{dy}\right) + (x^2 - xy) = 0$$

$$\left(\frac{x}{y} + 1\right)\left(v + y\frac{dv}{dy}\right) + \left(\left(\frac{x}{y}\right)^2 - \frac{x}{y}\right) = 0$$

$$(v + 1)\left(v + y\frac{dv}{dy}\right) + (v^2 - v) = 0$$

$$(v + 1)v + (v + 1)y\frac{dv}{dy} + (v^2 - v) = 0$$

$$(v + 1)y\frac{dv}{dy} = -2v^2$$

$$\int \frac{(v + 1)}{v^2}\,dv = -2\int \frac{1}{y}\,dy$$

$$\ln v - \frac{1}{v} = -2\ln y + c$$

$$\ln v - \frac{1}{v} = -\ln y^2 + c$$

$$\ln vy^2 = \frac{1}{v} + c$$

$$\ln \frac{x}{y}y^2 = \frac{1}{\frac{x}{y}} + c$$

$$\ln xy = \frac{y}{x} + c$$

Method 2

$$(xy + y^2) + (x^2 - xy)\frac{dy}{dx} = 0$$

Let $u = {}^{y}/_{x}$

$\therefore y = ux$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$\left(\frac{y}{x} + \frac{y^2}{x^2}\right) + \left(1 - \frac{y}{x}\right)\left(u + x\frac{du}{dx}\right) = 0$$

$$(u + u^2) + (1 - u)\left(u + x\frac{du}{dx}\right) = 0$$

$$(u + u^2) + (1 - u)u + (1 - u)x\left(\frac{du}{dx}\right) = 0$$

$$2u + (1 - u)x\left(\frac{du}{dx}\right) = 0$$

$$\int \frac{2}{x}\,dx = -\int \frac{(1 - u)}{u}\,du + c$$

$$2\ln x = -\ln u + u + c$$

$$2\ln x = -\ln\frac{y}{x} + \frac{y}{x} + c$$

$$\ln x^2 + \ln\frac{y}{x} = \frac{y}{x} + c$$

$$\ln\frac{x^2 y}{x} = \frac{y}{x} + c$$

$$\ln xy = \frac{y}{x} + c$$

d) **Method I**

$$(x^3 + 3xy^2)\frac{dy}{dx} = y^3 + 3x^2y$$

Let $v = {}^{x}/_{y}$

$$\therefore \ \frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$(x^3 + 3xy^2) = (y^3 + 3x^2y)\frac{dx}{dy}$$

$$\left(\frac{x^3}{y^3} + 3\frac{x}{y}\right) = \left(1 + 3\frac{x^2}{y^2}\right)\left(v + y\frac{dv}{dy}\right)$$

$$(v^3 + 3v) = (1 + 3v^2)\left(v + y\frac{dv}{dy}\right)$$

$$v^3 + 3v = v + 3v^3 + y(1 + 3v^2)\frac{dv}{dy}$$

$$-2v^3 + 2v = y(1 + 3v^2)\frac{dv}{dy}$$

$$2\int \frac{1}{y}dy = \int \frac{1 + 3v^2}{v(1 - v^2)}dv$$

$$\frac{1 + 3v^2}{v(1 - v^2)} = \frac{A}{v} + \frac{B}{(1 - v)} + \frac{C}{(1 + v)}$$

$$1 + 3v^2 = A(1 - v^2) + Bv(1 + v) + Cv(1 - v)$$

$$A = 1, B = 2, C = -2$$

$$2\int \frac{1}{y}dy = \int \frac{dv}{v} + 2\int \frac{dv}{(1 - v)} - 2\int \frac{dv}{(1 + v)}$$

$$2\ln y = \ln v - 2\ln(1 - v) - 2\ln(1 + v) + \ln c$$

$$= \ln v - 2\ln(1 - v^2) + \ln c$$

$$\ln y^2 = \ln\frac{v}{(1 - v^2)^2} + \ln c$$

$$\ln y^2 c = \ln\frac{v}{(1 - v^2)^2}$$

$$y^2 c = \frac{v}{(1 - v^2)^2}$$

$$y^2 c = \frac{\frac{x}{y}}{\left(1 - \frac{x^2}{y^2}\right)^2}$$

$$\frac{xy}{(x^2 - y^2)^2} = c$$

Method 2

$$(x^3 + 3xy^2)\frac{dy}{dx} = y^3 + 3x^2 y$$

Let $u = \dfrac{y}{x}$

$$\therefore \quad \frac{dy}{dx} = u + x\frac{du}{dx}$$

$$\left[1 + 3\left(\frac{y}{x}\right)^2\right]\frac{dy}{dx} = \left(\frac{y}{x}\right)^3 + 3\frac{y}{x}$$

$$(1 + 3u^2)\left(u + x\frac{du}{dx}\right) = u^3 + 3u$$

$$(1 + 3u^2)x\frac{du}{dx} = u^3 + 3u - u(1 + 3u^2)$$

$$(1 + 3u^2)x\frac{du}{dx} = 2(u - u^3)$$

$$\int \frac{(1 + 3u^2)}{(u - u^3)}du = \int \frac{2}{x}dx$$

$$\int \frac{(1 + 3u^2)}{u(1 - u)(1 + u)}du = \int \frac{2}{x}dx$$

$$\int \left(\frac{1}{u} + \frac{2}{1 - u} - \frac{2}{1 + u}\right)du = \int \frac{2}{x}dx$$

$$\ln u + 2\ln(1 - u) - \ln(1 + u) = 2\ln x + \ln c$$

$$\ln\frac{u}{(1 - u^2)} = \ln x^2 c$$

$$\frac{u}{(1 - u^2)} = x^2 c$$

$$\frac{\frac{y}{x}}{\left(1-\left(\frac{y}{x}\right)^2\right)} = x^2 c$$

$$\frac{xy}{(x^2-y^2)^2} = c$$

e)

$$(x-y-1)+(4y+x-1)\frac{dy}{dx} = 0$$

Let $x = X + a \qquad y = Y + b$

$x - y - 1 = X + a - Y - b - 1$

$\qquad = X - Y \quad$ when $\quad a - b - 1 = 0 \quad --- (1)$

$4y + x - 1 = 4(Y + b) + (X + a) - 1$

$\qquad = 4Y + X \quad$ when $\quad 4b + a - 1 = 0 --- (2)$

$(2) - (1) \qquad 5b = 0$

$\qquad b = 0$

$\qquad a = 1$

$$(x-y-1)+(4y+x-1)\frac{dy}{dx} = 0$$

$$(X-Y)+(4Y+X)\frac{dY}{dX} = 0$$

Let $V = \dfrac{Y}{X} \qquad \therefore \; Y = VX$

$$\therefore \; \frac{dY}{dX} = V + X\frac{dV}{dX}$$

$$(1-V)+(4V+1)\left[V + X\frac{dV}{dX}\right] = 0$$

$$1 - V + 4V^2 + V + X(4V+1)\frac{dV}{dX} = 0$$

$$(4V^2+1) + X(4V+1)\frac{dV}{dX} = 0$$

$$\int \frac{(4V+1)}{(4V^2+1)} dV + \int \frac{1}{X} dX = 0$$

$$\frac{1}{2}\int \frac{8V}{4V^2+1}\,dV + \frac{1}{4}\int \frac{1}{V^2+\left(\frac{1}{2}\right)^2}\,dV + \int \frac{1}{X}\,dX = 0$$

$$\frac{1}{2}\ln(4V^2+1) + \frac{1}{4}\times\frac{1}{1/2}\tan^{-1}\frac{V}{1/2} + \ln X = c$$

$$\frac{1}{2}\ln(4V^2+1) + \frac{1}{2}\tan^{-1}2V + \ln X = c$$

$$\frac{1}{2}\ln\left(4\frac{Y^2}{X^2}+1\right) + \frac{1}{2}\tan^{-1}\frac{2Y}{X} + \ln X = c$$

$$\frac{1}{2}\ln\left(\frac{4Y^2+X^2}{X^2}\right) + \frac{1}{2}\tan^{-1}\frac{2Y}{X} + \ln X = c$$

$$\frac{1}{2}\ln\left(\frac{4y^2+(x-1)^2}{(x-1)^2}\right) + \frac{1}{2}\tan^{-1}\frac{2y}{(x-1)} + \ln(x-1) = c$$

2. a)

$$(1+x^2)\frac{dy}{dx} + 3xy = 5x$$

$$\frac{dy}{dx} + \left(\frac{3xy}{1+x^2}\right) = \left(\frac{5x}{1+x^2}\right) --- (1)$$

$$I.F. = e^{\int\left(\frac{3x}{1+x^2}\right)dx} = e^{\frac{3}{2}\int\left(\frac{2x}{1+x^2}\right)dx} = e^{\frac{3}{2}\ln\left(1+x^2\right)} = e^{\frac{3}{2}\ln\left(1+x^2\right)} = e^{\ln\left(1+x^2\right)^{\frac{3}{2}}}$$

$$= (1+x^2)^{\frac{3}{2}}$$

$$(1)\times(1+x^2)^{\frac{3}{2}}$$

$$(1+x^2)^{\frac{3}{2}}\frac{dy}{dx} + 3x(1+x^2)^{\frac{1}{2}}y = 5x(1+x^2)^{\frac{1}{2}}$$

$$\frac{d\left\{(1+x^2)^{\frac{3}{2}}y\right\}}{dx} = 5x(1+x^2)^{\frac{1}{2}}$$

$$(1+x^2)^{\frac{3}{2}}y = \int 5x(1+x^2)^{\frac{1}{2}}$$

$$= \frac{5}{2}\int 2x(1+x^2)^{\frac{1}{2}}\,dx$$

$$= \frac{5}{2} \; \frac{1}{\left(\frac{1}{2}+1\right)} (1+x^2)^{\frac{3}{2}} + c$$

$$(1+x^2)^{\frac{3}{2}}\, y = \frac{5}{3} \; (1+x^2)^{\frac{3}{2}} + c$$

When $x = 1, y = 2$

$$(2)^{\frac{3}{2}}\, 2 = \frac{5}{3} \; (2)^{\frac{3}{2}} + c$$

$$c = \frac{1}{3} 2^{\frac{3}{2}}$$

$$(1+x^2)^{\frac{3}{2}}\, y = \frac{5}{3} \; (1+x^2)^{\frac{3}{2}} + \frac{1}{3} 2^{\frac{3}{2}}$$

b)

$$\frac{dy}{dx} + y \cot x = 5\, e^{\cos x}$$

$$I.F. = e^{\int \cot x \, dx} = e^{\int \frac{\cos x}{\sin x} dx} = e^{\ln|\sin x|} = \sin x$$

$$\therefore \sin x \frac{dy}{dx} + y \cos x = 5 \sin x \;\, e^{\cos x}$$

$$\frac{d\,(y \sin x)}{dx} = 5\, e^{\cos x} \; \sin x$$

$$y \sin x \; = \int 5\, e^{\cos x} \; \sin x + c$$

$$y \sin x \; = -5\, e^{\cos x} \; + c$$

When $x = \frac{\pi}{2}, y = -4$

$-4 \; = -5 \; + c \qquad\qquad \therefore c = 1$

$$y \sin x \; = -5\, e^{\cos x} \; + 1$$

3. a)

$$EI\frac{d^2y}{dx^2} = \frac{\omega}{2}(l-x)^2$$

$$\frac{2EI}{\omega}\ \frac{d^2y}{dx^2} = (l-x)^2$$

$$\frac{2EI}{\omega}\ \frac{dy}{dx} = \int (l-x)^2\,dx$$

$$\frac{2EI}{\omega}\ \frac{dy}{dx} = \frac{-1}{3}(l-x)^3 + A - - - (1)$$

$$\frac{2EI}{\omega}\ y = \int \frac{-1}{3}(l-x)^3 dx + \int A\,dx$$

$$\frac{2EI}{\omega}\ y = \frac{1}{3}\times\frac{1}{4}(l-x)^4 + Ax + B - - - (2)$$

When $x = 0$, $\quad \frac{dy}{dx} = 0$

From (1)

$$\frac{2EI}{\omega}\times 0 = -\frac{1}{3}l^3 + A \qquad \therefore A = \frac{1}{3}l^3$$

When $x = 0$, $y = 0$

From (2)

$$\frac{2EI}{\omega}\ \times 0 = \frac{1}{12}\ l^4 + 0\times x + B \quad \therefore B = -\frac{1}{12}\ l^4$$

$$\frac{2EI}{\omega}\ y = \frac{1}{12}(l-x)^4 + \frac{1}{3}l^3x - \frac{1}{12}\ l^4$$

When $x = l$,

$$\frac{2EI}{\omega}\ y = \frac{1}{3}l^4 - \frac{1}{12}\ l^4 = \frac{3}{12}l^4$$

$$y = \frac{\omega}{2EI}\left(\frac{1}{3}l^4 - \frac{1}{12}\ l^4\right) = \frac{l^4\omega}{8EI}$$

b)

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

Auxiliary equation

$m^2 + 5m + 6 = 0$

$(m+3)(m+2) = 0$

$m = -3$ & $m = -2$

$x = Ae^{-3t} + Be^{-2t} --- (1)$

$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} --- (2)$

When $t = 0$, $x = 0.1$ By (1) $0.1 = A + B$

$\frac{dx}{dt} = 0$ By (2) $0 = 3A + 2B$

$A = -0.2$ & $B = 0.3$

$x = 0.2e^{-3t} - 0.3e^{-2t}$

4. a) $y_s = y_0 - \Delta y_0 \binom{s}{1} + \Delta^2 y_0 \binom{s}{2} + \Delta^3 y_0 \binom{s}{3} + \dots\dots + \Delta^n y_0 \binom{s}{n}$

Where

$\binom{s}{r} = \frac{s(s-1)(s-2)\dots\dots..(s+1-r)}{r!}$

b)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.00	1.00						
		0.0247					
1.05	1.024695		-0.00059				
		0.02411		0.00005			
1.10	1.048809		-0.00054		-0.00002		
		0.02357		0.00003		0.00003	
1.15	1.072381		-0.00051		0.00001		-0.00006
		0.02306		0.00004		-0.00003	
1.20	1.095445		-0.00047		-0.00002		
		0.02259		0.00002			
1.25	1.118034		-0.00045				
		0.02214					
1.30	1.14017						

c) By using Newton's backward difference interpolation formula

$$y_s = y_7 - \Delta y_6 \binom{s}{1} + \Delta^2 y_5 \binom{s}{2} - \Delta^3 y_4 \binom{s}{3} + \Delta^4 y_3 \binom{s}{4} - \Delta^5 y_2 \binom{s}{5} + \Delta^6 y_1 \binom{s}{6}$$

$$s = \frac{(x_7 - x)}{h} = \frac{130 - 1.28}{0.05} = 0.4$$

$$\binom{s}{1} = 0.4$$

$$\binom{s}{2} = \frac{0.4(-0.6)}{2!} = -0.12$$

$$\binom{s}{3} = \frac{0.4(-0.6)(-1.6)}{3!} = 0.064$$

$$\binom{s}{4} = \frac{0.4(-0.6)(-1.6)(-2.6)}{4!} = -0.0416$$

$$\binom{s}{5} = \frac{0.4(-0.6)(-1.6)(-2.6)(-3.6)}{5!} = 0.0230$$

$$\binom{s}{6} = \frac{0.4(-0.6)(-1.6)(-2.6)(-3.6)(-4.6)}{6!} = -0.030$$

$$y_s = 1.14017 - (0.02214 \times 0.4) + (0.00045 \times -0.12) - (0.00002 \times 0.064)$$

$$+(0.00002 \times -0.0416) - (-0.00003 \times 0.030) + (-0.00006 \times -0.023)$$

$$= 1.131375$$

5. a)

i	0	1	2	3	4	5
x_i	1980	1985	1990	1995	2000	2005
s_i	0	1	2	3	4	5
$y_i = f(x_i)$	440	510	525	571	500	600

$$\pi(s) = s(s-1)(s-2)(s-3)(s-4)(s-5)$$

$$= s(s-1)(s^2 - 5s + 6)(s^2 - 9s + 20)$$

$$= (s^2 - s)(s^4 - 14s^3 + 71s^2 - 154s + 120)$$

$$= s^6 - 15s^5 + 85s^4 - 225s^3 + 274s^2 - 120s$$

$$L_0^5(s) = \frac{(s-1)(s-2)(s-3)(s-4)(s-5)}{(0-1)(0-2)(0-3)(0-4)(0-5)}$$

$$= \frac{s^5 - 15s^4 + 85s^3 - 225s^2 + 274s - 120}{-120}$$

$$L_1^5(s) = \frac{s(s-2)(s-3)(s-4)(s-5)}{(1-0)(1-2)(1-3)(1-4)(1-5)}$$

$$= \frac{s^5 - 14s^4 + 71s^3 - 154s^2 + 120s}{24}$$

$$L_2^5(s) = \frac{s(s-1)(s-3)(s-4)(s-5)}{(2-0)(2-1)(2-3)(2-4)(2-5)}$$

$$= \frac{s^5 - 13s^4 + 59s^3 - 107s^2 + 60s}{-12}$$

$$L_3^5(s) = \frac{s(s-1)(s-2)(s-4)(s-5)}{(3-0)(3-1)(3-2)(3-4)(3-5)}$$

$$= \frac{s^5 - 12s^4 + 49s^3 - 78s^2 + 40s}{12}$$

$$L_4^5(s) = \frac{s(s-1)(s-2)(s-3)(s-5)}{(4-0)(4-1)(4-2)(4-3)(4-5)}$$

$$= \frac{s^5 - 11s^4 + 36s^3 - 36s^2 + 30s}{-24}$$

$$L_5^5(s) = \frac{s(s-1)(s-2)(s-3)(s-4)}{(5-0)(5-1)(5-2)(5-3)(5-4)}$$

$$= \frac{s^5 - 10s^4 + 30s^3 - 30s^2 + 24s}{120}$$

Where

$$s = \frac{x - 1980}{5}$$

$$I(s) = y_0L_0^5(s) + y_1L_1^5(s) + y_2L_2^5(s) + y_3L_3^5(s) + y_4L_4^5(s) + y_5L_5^5(s)$$

$$I(s) = y_0L_0^5(s) + y_1L_1^5(s) + y_2L_2^5(s) + y_3L_3^5(s) + y_4L_4^5(s) + y_5L_5^5(s)$$

$$= 440 \times \left\{\frac{s^5 - 15s^4 + 85s^3 - 225s^2 + 274s - 120}{-120}\right\}$$

$$+ 510 \times \left\{\frac{s^5 - 14s^4 + 71s^3 - 154s^2 + 120s}{24}\right\}$$

$$+ 525 \times \left\{\frac{s^5 - 13s^4 + 59s^3 - 107s^2 + 60s}{-12}\right\}$$

$$+ 571 \times \left\{\frac{s^5 - 12s^4 + 49s^3 - 78s^2 + 40s}{12}\right\}$$

$$+ 500 \times \left\{\frac{s^5 - 11s^4 + 36s^3 - 36s^2 + 30s}{-24}\right\}$$

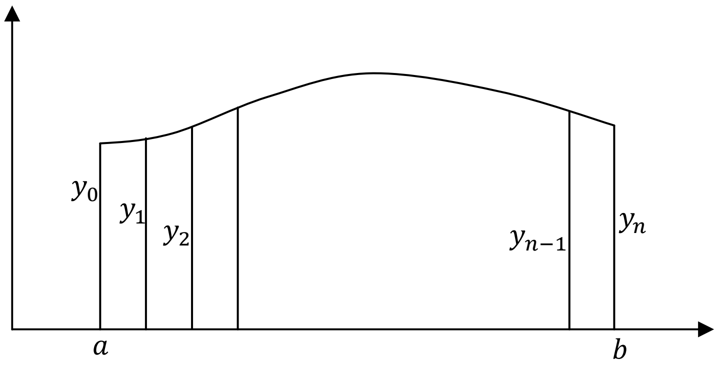
$$+ 600 \times \left\{\frac{s^5 - 10s^4 + 30s^3 - 30s^2 + 24s}{120}\right\}$$

$x = 1998 \quad \therefore s = \dfrac{1998 - 1980}{5} = \dfrac{18}{5} = 3.6$

$I(s) = 541.5785$

b)

i)



$\Delta =$ Area of the curve between $x = a$ and $x = b$

$h = \dfrac{b-a}{n} \qquad n -$ number of strips

$$\Delta = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \cdots \ldots + y_{n-1})]$$

ii)

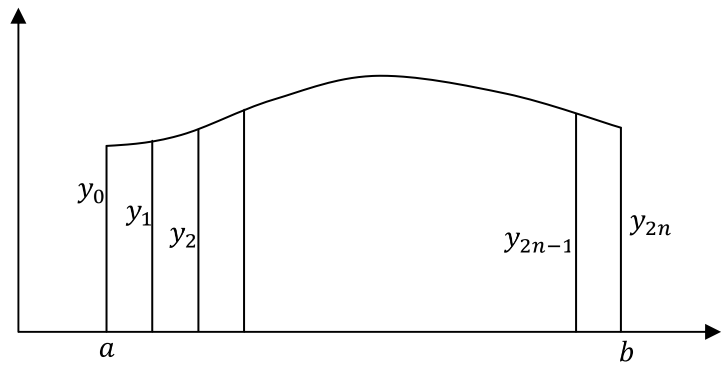
$$f = \frac{dv}{dt}$$

$$v = \int_0^{60} f\, dt$$

$$\Delta = \frac{20}{2}[(57.5 + 67.2) + 2(59.0 + 63.8)]$$

$$\Delta = 3703\, ms^{-1}$$

6. a)



$$\int_a^b f(x)dx = \frac{h}{3}\left[(y_0 + y_{2n}) + 4(y_1 + y_3 + \cdots + y_{2n-1}) + 2(y_2 + y_4 + \cdots + y_{2n-2})\right]$$

$h = \dfrac{b-a}{2n}$ (with even number of strips)

b)

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	0.0000	0.1679	0.2955	0.4118	0.5460
$(1+x)^2$	0	1.44	1.96	2.56	3.24
$(1+x)^2 f(x)$	0	0.2418	0.5870	1.0542	1.7693
$[f(x)]^2$	0	0.0282	0.0897	0.1696	0.2981

i) $$\int_0^{0.8} (1+x)^2 f(x)\,dx = \frac{0.2}{3}\left[(0 + 1.7690) + 4(0.2418 + 1.0542) + 2(0.5870)\right]$$

$$\int_0^{0.8} (1+x)^2 f(x)\,dx = 0.5407$$

ii) $$\int_0^{0.8} [f(x)]^2\,dx = \frac{0.2}{3}\left[(0 + 0.2981) + 4(0.0282 + 0.1696) + 2(0.0897)\right]$$

$$\int_0^{0.8} [f(x)]^2\,dx = 0.0842$$